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Fractal Structure of Quantum Gravity and Relic Radiation Anisotropy

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It is argued that the large-scale (>7°) cosmic microwave background anisotropy detected in the COBE cosmic experiment can be considered as a trace of the fractal structure of quantum gravity.

1. INTRODUCTION

The universe seems to exhibit a fractal structure. Superclusters (large clusters containing up to hundreds of thousands of galaxies) of size about 50 Mpc are separated by almost empty space: the mean distance between two superclusters is about 100 Mpc. Clusters of galaxies (a typical cluster size is about 5 Mpc) containing hundreds of galaxies are, in turn, separated by voids of about a few Mpc. This fractal hierarchy can be easily traced to subnuclear scales (10^{-13} cm). Quantitatively, the large-scale fractal structure of the universe can be described in terms of the mass interior to a spherical volume of a certain radius r. A typical dependence measured by observing the 21-cm hydrogen emission of gas clouds moving around the galaxy is

$$\mathcal{M}(r) \propto r^{\alpha}, \qquad \alpha \approx 1 \tag{1}$$

whereas a luminous mass associated with the light would yield only $r^{-1/2}$. It is commonly accepted that additional mass is present in the form of nonluminous dark matter (Sancisi and van Albada, 1987). Since $\alpha < 3$ in the power law (1), we have a typical mass distribution on a fractal set embedded in $D = 3$ space.

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On the other hand, one of the most important recent developments in gravitational theory is related to the fractal-based regularization of quantum gravity (Knizhnik *et al.*, 1988). In view of this one may believe that a fractal structure is a fundamental property of physical space-time itself.

In this paper we interpret the COBE satellite data on the anisotropy of the cosmic microwave background radiation (CMBR) as a possible manifestation of the fractal structure of the universe.

2. ON DISCRETE SYMMETRY IN QUANTUM GRAVITY

The regularization of two-dimensional quantum gravity by Knizhnik, Polyakov, and Zamolodchikov (1988) (KPZ) follows from the fact that the continuum formulation (Polyakov, 1987) and the dynamical triangulation (Boulatov *et al.*, 1986) are equivalent. On the basis of the Polyakov (1981) regularization procedure, where the position of the surface in the embedding space X_μ and the internal surface geometry (g_{ab}) are treated as independent fields, one can construct a Nambu-like action

$$\begin{aligned} S[X_\mu, g_{ab}] &= \frac{1}{2}\int_M g_{ab}\frac{\partial X_\mu}{\partial \xi_a}\frac{\partial X_\mu}{\partial \xi_b}(\det g)^{1/2}\, d^2\xi + \beta\int_M (\det g)^{1/2}\, d^2\xi \\ &\quad + \text{fermion terms} \end{aligned} \tag{2}$$

where $\xi = (\xi_1, \xi_2)$ is the parametrization of the manifold M defined by function $X_\mu = X_\mu(\xi)$. This or a similar Nambu–Goto action usually appears in the string functional integral taken with respect to both independent fields X and g.

In the dynamical triangulation (Boulatov *et al.*, 1986) of 2D quantum gravity, as well as in higher dimensional versions (Ambjørn *et al.*, 1993), the path integral over the internal metric g_{ab} is replaced by summation of all the different types of surface configurations with a given number of triangles. For the sake of preserving reparametrization invariance after discretization (Boulatov *et al.*, 1986), the topology of the manifold M is usually specified as the sphere S^2 (Boulatov *et al.*, 1986; Kawamoto, 1993). The partition function takes the form

$$Z(A) = \int_M \mathcal{D}X\,\mathcal{D}g\,\exp(-S) \tag{3}$$

or its discrete counterpart (Kawamoto, 1993)

$$Z_{\text{reg}}(A) = \sum_{G} Z_m(G)\delta_{Na^2,A} \tag{4}$$

where A is the total area, N is the number of equilateral triangles, and a^2 is the area of a triangle. The matter part of the partition function $Z_m(G)$ comes from the fermion term of the KPZ Lagrangian

$$\mathcal{L} = \bar{\phi} v^{\alpha a} \gamma^a \partial_\alpha \phi \tag{5}$$

where $v_{\alpha a}$ are ordinary "zweibeins."

Formally substituting for the functional integral (3) its discrete counterpart (4), we need to sum over all possible triangulations of S^2. Practically, we are to impose some additional conditions to avoid summation over singular triangulations, i.e., triangulations which include links with coinciding ends. Referring the reader to Boulatov *et al.* (1986) and Kawamoto *et al.* (1992) for detailed study of triangulations and fractal properties of related partition functions, we shall concentrate on some of their properties significant for phenomenological applications.

1. The triangulation procedure can be extended to an S^n sphere (Ambjørn *et al.*, 1993), which, as a boundary of an $(n + 1)$-dimensional simplex, can be divided into n-dimensional simplices.

2. From the conformal invariance standpoint, of all the subdivisions of S^n, the subdivision into equilateral simplices is preferable.

3. The whole partition function (3) is related to a physical object which is isotropic (in the sense of having no preferable direction on S^n), but may have a discrete symmetry group and hence have certain distinguished correlation angles. For example, if we sum over all possible triangulations of S^2 using equilateral triangles, the correlations of any observables depending on matter fields increase at angles 0, $2\pi/3$, $4\pi/3$ because of the Z_3 symmetry group. Similarly, the correlations should increase at tetrahedron group angles when S^3 is considered.

4. Two-dimensional quantum gravity can be regarded as only the simplest case of extended-object physics. However, when reducing the physics from arbitrary n-dimensional space to $n - 1$ dimensions we restrict S^n triangulation with n-dimensional simplices to S^{n-1} triangulation with $(n - 1)$-dimensional ones, because an $(n - 1)$-dimensional simplex is a boundary of an n-dimensional one. Thus, for the case of equilateral simplices we should always have Z_3-symmetry in $D = 2$ or tetrahedron symmetry in $D = 3$.

3. DISCRETE SYMMETRY AS A POSSIBLE SOURCE OF RELIC RADIATION ANISOTROPY

Let us consider the data on relic radiation anisotropy (Smoot *et al.*, 1992). The relic microwave radiation ($T = 2.73$ K) has not been significantly affected by late-stage processes in the universe, which is why its amplitudes depend mostly on the parameters of the early universe. It is worth noting that the large-scale anisotropy of relic radiation found in COBE and RELICT-1 experiments has a rather small value, $\Delta T/T \sim 10^{-5}$, but a high confidence level—up to 90%, including systematic errors (Smoot *et al.*, 1992; Strukov *et al.*, 1993).

The first aim of the observers in both COBE and RELICT experiments was to measure the dipole and quadrupole components of the microwave background (Smoot *et al.*, 1992) and to test for the existence of an anomalous signal over the mean background (Strukov *et al.*, 1993). Based on the COBE experiment data, the autocorrelation function

$$C(\alpha) = \langle \Delta T(\theta) \Delta T(\theta + \alpha) \rangle \tag{6}$$

has been obtained. Here α is the angular separation and θ is an angular coordinate on a certain two-dimensional plane.

Qualitatively, the behavior of the relic signal autocorrelation function (see Fig. 1) is the following: it has a sharp maximum, it has another maximum localized at α close to 120°, and it has two minima at 60° and 180° (the maximum at α close to 90° is possibly related to a quadrupole component of CMBR and is less confident (Bennet *et al.*, 1992)). The behavior of the

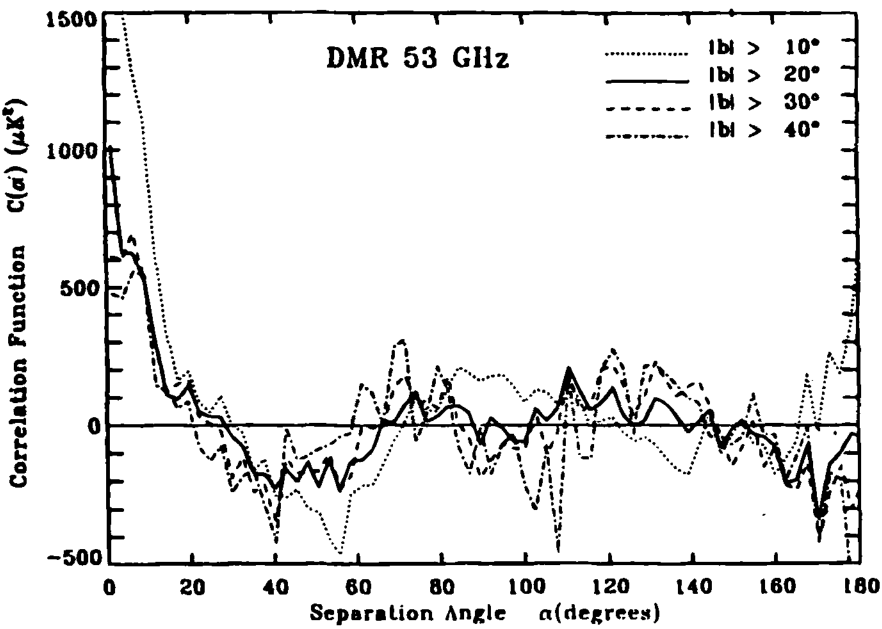

Fig. 1. Correlation function $C(\alpha)$ at various galactic latitude cuts for the 53-MHz map. (Reprinted from Smoot *et al.*, 1992.)

autocorrelation functions is almost the same for the data obtained at frequencies 53 and 90 GHz (Smoot *et al.*, 1992).

The correlation function (6) was studied in Wright *et al.* (1992) in connection with present cosmological models. In particular, an attempt was made to compare the COBE data with certain dark matter (DM) models. This comparison did not go well. For instance, the relic density anisotropy given by the Holtzman (1989) model increases monotonously with α increasing from 60° to 180° (Wright *et al.*, 1992).

Taking into account all the mentioned arguments, we interpret the regularities of the behavior of the autocorrelation function (6) as a manifestation of Z_3-symmetry. The presence of Z_3-symmetry does not imply n preferable directions in space here; instead we have a preferred separation angle. It should be mentioned that in the theoretical COBE study of Wright *et al.* (1992) the best-line fit for the autocorrelation function (6) was taken in the form

$$C(\alpha) = A + B \cos \alpha + C_M^0 \exp\left(-\frac{\alpha^2}{2\sigma^2}\right) \tag{7}$$

although the locations of autocorrelation function maximas at 0° and 120° and minimas at 60° and 180° suggest the more direct parametrization

$$C(\alpha) = A + B \cos 3\alpha + C_M^0 \exp\left(-\frac{\alpha^2}{2\sigma^2}\right) \tag{8}$$

4. ON THE FLAT-SPACE LIMIT OF SIMPLICIAL QUANTUM GRAVITY

The simplest way to imagine how the distribution of relic radiation with simplicial symmetry could emerge from space-time geometry is by considering simplicial quantum gravity (Ambjørn and Jurkiewicz, 1992; Agishtein and Migdal, 1992). This theory enables one to describe only pure gravity without matter fields in a consistent way. An exact solution for matter coupling has been found only for the two-dimensional case (Kazakov *et al.*, 1985; Boulatov *et al.*, 1986). For higher dimensions, if we want to describe nature as it is, we have to face a lot of matter coupling problems. The question of our particular concern should be the existence of the flat-space continuous limit. Here we analyze problems arising in the $D > 2$ continuous limit of simplicial quantum gravity and suggest a way to avoid them.

The very fact which, in our opinion, led to the KPZ regularization of two-dimensional quantum gravity was the fractal nature of a dynamically triangulated surface, rather than the simplicial structure itself. [An investigation in some way similar has been performed by Crane and Smolin (1986) without using triangulation at all.] That is why we should expect some fractal structure which enables us to remove the divergences.

Indeed, in direct studies of quantum field theory models on fractal spacetime (Eyink, 1989a,b) as well as in studies devoted to fractal lattices (Gefen *et al.*, 1983a,b) it has been shown that fractal sets, being scale invariant, are essential for using the renormalization group (RG) technique. Unfortunately, the price for well-defined scale properties is the lack of translation invariance. This leads to the divergences. Until now this obstacle has not been completely overcome.

As we consider the flat-space limit of (Euclidean) simplicial gravity, we must pay attention to those fractal sets which are suitable for triangulation of an S^n sphere. Thus, we consider the *Sierpinski hypergasket*, a generalization of the Sierpinski gasket constructed in two dimensions. Let us recall the construction procedure (Eyink, 1989a,b).

Partitioning the unit d-simplex in R^d into $(d + 2)$ subsimplices of edge length 1/2, one (i) removes the open central subsimplex and (ii) repeats the operation with the $(d + 1)$ closed subsimplices.

Sierpinski gaskets obtained in this way can be used for triangulation of an S^n-sphere. Their self-similarity is very relevant to RG applications. Their shortcomings are also evident. They are not invariant under translations, even inside a single gasket, and they are not dense in the embedding space. That is why we seek a better simplicial fractal set in this regard.

Let us modify the gasket generating procedure. To clarify considerations, let us imagine a black unit simplex. On the first step of the recursive procedure we remove its central open part; the central subsimplex becomes white, and then—here is the difference—we repeat the procedure with *all* $(d + 2)$ subsimplices. The generalization to white pieces seems evident: the central part of each simplex reverses its color. The difference between the Sierpinski gasket and our gasket is shown in gray in Fig. 2.

Since the numbers of black and white subsimplices at the $(k + 1)$ stage of the recursive procedure are

$$n_W^{k+1} = (d + 1)n_W^k + n_B^k$$
$$n_B^{k+1} = (d + 1)n_B^k + n_W^k \tag{9}$$

for asymptotically large k we obtain

$$n_k \approx \frac{1}{2}(d + 2)^k$$

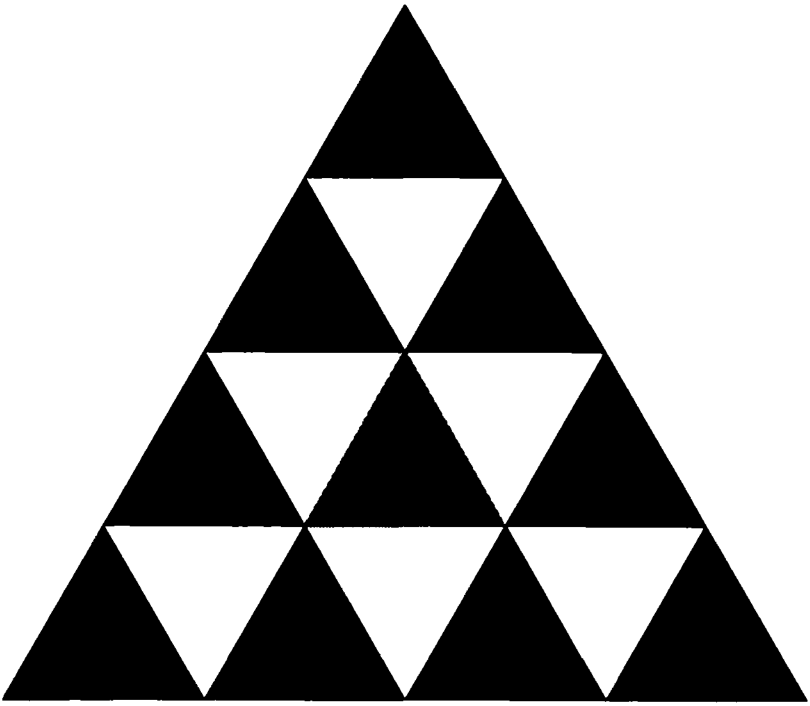

Fig. 2. Second stage of the (2D) fractal gasket construction. The Sierpinski gasket is shown in black. The new part appearing in the *yin-yang gasket* is shown in gray.

simplices of each color of $\delta = 2^{-k}$ edge size. The fractal dimension of the constructed set is

$$D = \frac{\log(d + 2)}{\log 2} \tag{10}$$

rather than

$$D = \frac{\log(d + 1)}{\log 2} \tag{11}$$

for the Sierpinski gasket, but the scaling law is identical:

$$2^k \cdot \mathcal{G}(d) = \mathcal{G}(d) \tag{12}$$

As far as we know, there is no commonly accepted name for such a set. Here, as the relation between black and white in the construction is much like the ancient Chinese yin and yang symbol, we can informally call it a yin-yang gasket.

The geometrical properties of the set constructed above as a building block for a piecewise approximation of an S^n sphere ($n > 2$) require further investigation. Nevertheless, we can already mention the properties which could be useful in simplicial quantum gravity phenomenology. The gasket is (i) *simplicial*, (ii) *scale-invariant*, (iii) *homogeneous*, and (iv) *dense in embedding space*.

5. CONCLUSION

The data on relic radiation anisotropy obtained by both the RELICT and COBE groups are worth further deep investigation. Even the results already obtained from the data seem to be in good agreement with the hypothesis of a discrete symmetry of space-time arising from fractal quantum gravity. Other cosmological data, e.g., mass distribution, do not contradict the possible fractal structure of the Universe. It might be argued that both the tetrahedron symmetry, if found, and the fractal structure of the visible universe can be regarded as an argument for the existence of cosmic strings (Kibble, 1976). Indeed, cosmic strings, as topological defects which could be formed at a phase transition in the early universe, can have a number of cosmological applications. In particular, they can form a network with a fractal structure having tetrahedron symmetry (Aryal *et al.*, 1986).

Therefore, some new tests for possible discrete symmetry can be proposed. The simplest among them are (i) to test cos $n\alpha$, $n > 1$, in (7) for other Z_n groups, and (ii) to use COBE and RELICT data to search for the tetrahedron or other essentially three-dimensional space symmetry groups.

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